EXAM COMPUTER VISION

November 8, 2004, 9:00 hrs

During the exam you may use the book, lab manual, copies of sheets and your own notes.

Put your name on all pages which you hand in, and number them. Write the total number of pages you hand in on the first page. Write clearly and not with pencil or red pen. ***Always motivate your answers.*** *Good luck!*

Problem 1. (2.5 pt) Consider finite, discrete, binary images X and Y, with 8-connected foreground (1) and 4-connected background (0) pixels.

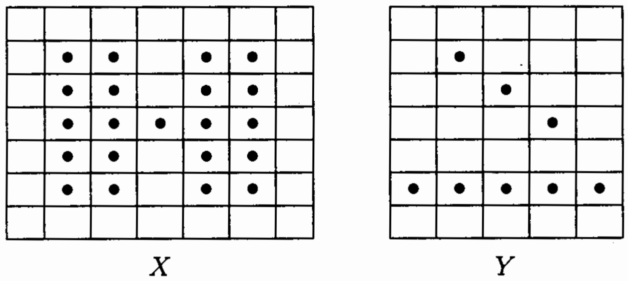

Figure 1: *Two binary images: squares connected by a bridge (left) and horizontal and diagonal lines (right)*

a. (1 pt) Suppose we want to detect foreground pixels which form a bridge between their right and left foreground neighbours, but do not have upper and lower foreground neighbours, using a hit-or-miss transform

$$\psi(X) := X \circledast (A^1, A^2).$$

Design a pair of structuring elements (A^1, A^2) which performs this task.

b. (0.5 pt) Check the result of ψ by drawing $\psi(X)$ and $\psi(Y)$ with X and Y as in Fig. 1.

c. (0.5 pt) Consider the transform $\bar{\psi}$ defined as

$$\bar{\psi}(X) = X \setminus \psi(X).$$

(Note \ is set difference). Show what this transform does to X and Y from Fig. 1, and what is the number of connected foreground components in X, $\bar{\psi}(X)$, Y, and $\bar{\psi}(Y)$.

d. (0.5 pt) Is ψ increasing? If not, give a counterexample.

Problem 2. (2 pt) Consider a spherical surface of radius r centered at the origin with equation

$$z = d - \sqrt{r^2 - x^2 - y^2}, \qquad x^2 + y^2 \leq r$$

The surface is Lambertian with constant albedo $\rho_S = 1$, and is illuminated by a light source at a very large distance, from a direction defined by the unit vector (a, b, c), with c negative. The camera is on the negative z-axis. Show that the image intensity under orthographic projection is given by

$$E(x, y) = \frac{ax + by - c\sqrt{r^2 - x^2 - y^2}}{r}$$

Problem 3. (2 pt) Consider a grey-value image f.

a. Sobel gradients in the image in horizontal (easterly) direction can be detected by linear filtering using the filter kernel (or mask) in Fig. 2 (left). Give Sobel kernels to detect gradients in northerly, north-westerly, and north-easterly direction.

b. A discrete second derivative filter in the x-direction $\frac{\partial^2}{\partial x^2}$ is defined by convolution with the kernel in Fig. 2(right). If image f is constant, the result of this filter will be zero in every pixel. Show by calculation that the result for an image $f(x, y) = ax^2 + bx + c$, is $-2a$ for each pixel with a, b, c constants.

-1	0	1
-2	0	2
-1	0	1

0	0	0
-1	2	-1
0	0	0

Figure 2: *Convolution masks for the Sobel x-gradient filter (left) and the second-order x-derivative filter (right).*

Problem 4. (2.5 pt) Consider the following inference problem. Given a perspective projection of a cube with three sets of four parallel ribs each, with unknown orientations $\vec{w}^{(X)}$, $\vec{w}^{(Y)}$ and $\vec{w}^{(Z)}$, and three corresponding vanishing points X, Y, Z in the projection plane, see Fig. 3. Two of these points are known $(u_\infty^{(Y)}, v_\infty^{(Y)}) = (1, 2)$, $(u_\infty^{(Z)}, v_\infty^{(Z)}) = (0, -2)$. The camera constant f is unknown.

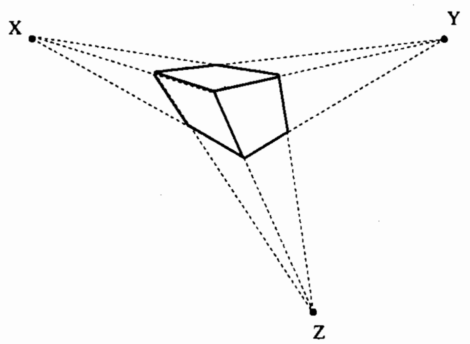

Figure 3: *Perspective projection of a cube with three vanishing points.*

Compute the three orientation vectors $\vec{w}^{(X)}, \vec{w}^{(Y)}, \vec{w}^{(Z)}$.
Hint: First compute the camera constant f.